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# Mass Matrix Textures From Superstring Inspired $SO(10)$ Models

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## Abstract

We give a general prescription for deriving quark and lepton mass matrices with “texture” zeros in the framework of superstring inspired  $SO(10)$  models. The key to our approach is a new way to naturally implement the doublet–triplet splitting which enables us to obtain symmetric quark and lepton mass matrices which have different structures in the up and the down quark sectors. We illustrate our method by deriving the Georgi–Jarlskog texture which has six predictions in the flavor sector, and then show how it generalizes to other symmetric texture models.

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One of the major challenges in particle physics today is an understanding of the spectrum of quarks and leptons. In the standard model, all the fermion masses and the mixing angles are arbitrary, accounting for 13 of the 19 free parameters of the theory. This situation is clearly unsatisfactory and has been viewed by many as a hint for physics beyond the standard model. A more fundamental theory, it is hoped, will reduce this arbitrariness, by providing a common origin perhaps for several of these parameters. In the absence of a fundamental theory of fermion masses, it has been popular to assume certain restricted forms for the quark and lepton mass matrices that results in predictions for some of the observables. The hope in this approach is that if the predictions of a particular mass matrix ansatz are borne out by experiments, then a model that leads to a natural derivation of that ansatz may provide the next step in our search for the fundamental theory of nature.

The next step beyond the standard model may very well contain a supersymmetric grand unified theory (SUSY GUT). This speculation is supported by the dramatically accurate unification of gauge couplings that has been observed to occur [1] with supersymmetry around the TeV scale following the improved measurements of the low energy couplings at LEP and SLC. Superstring models, which are candidate theories of unifying gravity with the strong and electroweak forces, can lead to a variety of such SUSY GUT groups via an appropriate compactification scheme. It therefore appears to us to be a very desirable program to see if a successful mass matrix ansatz can receive a plausible derivation within a SUSY GUT model inspired by superstrings. This is what we take up in this paper.

There are a variety of mass matrix ansatze which can be characterized by their “texture” zeros. For increased predictivity, it is usually assumed that the matrices are symmetric. A well-known example is the Fritzsch ansatz [2] which has a texture similar in form for the up and the down quarks. Such matrices are easily derived from GUTs such as  $SO(10)$ , which has built-in left-right symmetry and up-down symmetry. If the recent CDF data [3] on the top quark, viz.,  $m_t = 174 \pm 17 \text{ GeV}$ , holds up, it would appear that the Fritzsch ansatz would be ruled out even after renormalization group corrections are taken into account [4]. Another popular and predictive ansatz is the so-called Georgi–Jarlskog (GJ) texture [5], which assumes symmetric quark and lepton mass matrices, which however, have different forms in the up and down quark sectors. Such matrices, owing to their up-down asymmetry, are less trivial to derive in the context of SUSY GUTs such as  $SO(10)$ . In this letter, we provide a prescription for deriving such *up-down asymmetric* matrices in the context of superstring inspired  $SO(10)$  models. We will illustrate our method by deriving the GJ texture, which has been the subject of substantial improvement and polishing including renormalization group effects in recent literature [6,7,8,9,10]. We

will briefly summarize the six predictions of the ansatz to show its consistency with present data, especially with a heavy top as indicated by the recent CDF results [3]. Then we show how our method generalizes to other symmetric texture models. In particular, we show how it becomes possible to derive all of the five symmetric models listed in Ref. [11].

SUSY  $SO(10)$  seems to us to be the ideal setting for addressing the quark and lepton masses. All fermions of a family are unified to a single irreducible representation of  $SO(10)$ , which facilitates the generation of symmetric mass matrix textures. (This is not the case in  $SU(5)$  GUTs, where the down quark and the charged lepton mass matrices are not symmetric. Higher symmetries such as  $E_6$  invariably brings in exotic particles which can mix with the known quarks and leptons, thus generally reducing the predictive power.) The emergence of the right-handed neutrino in  $SO(10)$  leads to small but non-zero neutrino masses via the see-saw mechanism which may be highly desirable. The vexing problem of doublet-triplet splitting of SUSY GUTs also has an elegant resolution in  $SO(10)$ .

It has been known for some time that conventional SUSY GUTs such as  $SO(10)$  with massless matter superfields belonging to the adjoint representation (needed for gauge symmetry breaking) can arise in the free fermionic formulation of superstrings [12]. It is well-known that superstring theories restrict considerably the number and the nature of gauge multiplets that survive to low energies, thereby reducing the arbitrariness present in a general GUT. They also provide naturally the discrete symmetries which are often needed in restricting the texture of quark and lepton mass matrices. Specifically, we are encouraged by the recent works of Choudhuri, Chung and Lykken [13] and Cleaver [14] who have constructed explicit  $SO(10)$  models with adjoint scalars at the Kac-Moody level of two. These authors also classify the allowed representations that emerge as massless chiral multiplets below the Planck scale at the level two construction. While there can be any number of vectors (**10**), spinors ( $\overline{\mathbf{16}} + \mathbf{16}$ ) and gauge singlets, the number of adjoints is restricted to be at most 2. Similarly, no more than one **54** can remain light, although no explicit example with any **54** has been constructed so far.

We shall be guided in our derivation of the mass matrix “textures” by the superstring constraints listed above. Specifically, the Higgs representations that we shall use for symmetry breaking and for fermion mass generation will be a spinorial  $\overline{\mathbf{16}} + \mathbf{16}$ , a pair of **10**, two adjoint **45** and a few singlets. In order to make a realistic and predictive model for quarks and leptons at low energies, we shall impose the following requirements: (i) there must be a consistent mechanism to break the  $SO(10)$  symmetry down to the standard model at the GUT scale; (ii) the light particle spectrum of the theory must be such that it preserves the successful prediction for  $\sin^2\theta_W$ ; (iii) the doublet-triplet splitting should be achieved naturally (i.e., without

fine-tuning) in such a way that only one pair of Higgs doublet remains light (to be identified with  $H_u$  and  $H_d$  of MSSM); and finally, (iv) the same symmetry that helps satisfy the above requirements must provide an interesting texture for fermion mass matrices.

Let us first briefly summarize the predictions of the Georgi–Jarlskog ansatz. It assumes the following form for the up quark, down quark and the charged lepton mass matrices at the GUT scale:

$$M_u = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}; \quad M_d = \begin{pmatrix} 0 & de^{i\phi} & 0 \\ de^{-i\phi} & f & 0 \\ 0 & 0 & g \end{pmatrix}; \quad M_\ell = \begin{pmatrix} 0 & d & 0 \\ d & -3f & 0 \\ 0 & 0 & g \end{pmatrix}. \quad (1)$$

There are 7 parameters in all to fit the 13 observables (9 masses, 3 mixing angles and one CP phase), thereby resulting in six predictions. Three of these predictions are the  $b$ ,  $s$  and  $d$ -quark masses:

$$m_b = \eta_{b/\tau}^{-1} m_\tau; \quad \frac{m_d/m_s}{(1 - m_d/m_s)^2} = 9 \frac{m_e/m_\mu}{(1 - m_e/m_\mu)^2}; \quad (m_s - m_d) = \frac{1}{3} \eta_{s/\mu}^{-1} (m_\mu - m_e). \quad (2)$$

The other three predictions are for the quark mixing angles and the CP phase  $J$ :

$$|V_{cb}| = \eta_{KM}^{-1} \eta_{u/t}^{1/2} \sqrt{\frac{m_c}{m_t}}; \quad \frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{m_u}{m_c}}; \\ J = \eta_{KM}^{-2} \eta_{u/t} \sqrt{\frac{m_d}{m_s}} \sqrt{\frac{m_c}{m_t}} \sqrt{\frac{m_u}{m_t}} \left[ 1 - \frac{1}{4} \left( \sqrt{\frac{m_u}{m_c}} \sqrt{\frac{m_s}{m_d}} + \sqrt{\frac{m_c}{m_u}} \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_c}{m_u}} \sqrt{\frac{m_s}{m_d}} |V_{us}|^2 \right)^2 \right]^{\frac{1}{2}} \quad (3)$$

Here the  $\eta$ 's are renormalization factors for the various parameters in going from the low energy scale to the GUT scale. If the bottom-quark Yukawa coupling  $h_b$  is much smaller than the top Yukawa coupling  $h_t$ , (corresponding to  $\tan\beta \lesssim 10$  or so) these RGE factors can be expressed analytically as

$$\eta_{KM} = \eta_{d/b} = \left( 1 - \frac{Y_t}{Y_f} \right)^{\frac{1}{12}}; \quad \eta_{u/t} = \left( 1 - \frac{Y_t}{Y_f} \right)^{\frac{1}{4}}; \quad \eta_{s/\mu} = \left( \frac{\alpha_1}{\alpha_G} \right)^{-10/99} \left( \frac{\alpha_3}{\alpha_G} \right)^{-8/9}; \\ \eta_{b/\tau} = \left( \frac{\alpha_1}{\alpha_G} \right)^{-10/99} \left( \frac{\alpha_3}{\alpha_G} \right)^{-8/9} \left( 1 - \frac{Y_t}{Y_f} \right)^{-1/12}. \quad (4)$$

Here  $\alpha_G$  is the unified gauge coupling strength,  $Y_t = h_t^2$  at the weak scale and  $Y_f$  is the fixed point value of  $Y_t$ .  $Y_t$  cannot strictly be equal to  $Y_f$ , since that would correspond to infinite  $Y_t$  at the GUT scale. If we demand that  $Y_t \lesssim 4$  at GUT scale,  $Y_t/Y_f$  can at most be 0.98. The renormalization factors in Eq. (4) are all well-behaved even when  $Y_t$  differs from  $Y_f$  by only 2% due to the small exponents.

In the three Yukawa unification scheme ( $h_t = h_b = h_\tau$  at the GUT scale), the exponents  $(1/12, 1/12, 1/4)$  corresponding to  $(\eta_{KM}, \eta_{d/b}, \eta_{u/t})$  in Eq. (4) will be replaced by  $(1/7, 2/7, 2/7)$ ;  $\eta_{s/\mu}$  will remain unchanged, while  $\eta_{b/\tau}$  is modified to

$$\eta_{b/\tau} = \left(\frac{\alpha_1}{\alpha_G}\right)^{-19/376} \left(\frac{\alpha_3}{\alpha_G}\right)^{-2/9} \left(\frac{Y_t}{Y_\tau}\right)^{-3/8} \left(1 - \frac{Y_t}{Y_f}\right)^{-1/14}. \quad (5)$$

The  $\eta$  factors given in Eqs. (2)-(5) correspond to running the parameters from the weak scale (taken here to be  $m_t$  which is also assumed to be the SUSY threshold) up to  $M_{GUT}$ . For the light fermion masses there is QCD and QED running factors below  $m_t$  as well. These are obtained numerically using three loop QCD and one-loop QED  $\beta$  and  $\gamma$  functions. Corresponding to  $\alpha_s(M_Z) = 0.12$  and  $\alpha^{-1}(M_Z) = 127.9$ , these factors are  $(\eta_u, \eta_{d,s}, \eta_c, \eta_b, \eta_{e,\mu}, \eta_\tau) = (0.401, 0.404, 0.460, 0.646, 0.982, 0.984)$ . If  $\alpha_s(M_Z) = 0.125$  is chosen these factors become  $(0.356, 0.358, 0.422, 0.630, 0.982, 0.984)$ . Using  $\alpha_s(M_Z) = 0.12$ ,  $m_c(m_c) = 1.27 \text{ GeV}$ ,  $m_u(1 \text{ GeV}) = 5.1 \text{ MeV}$ ,  $|V_{us}| = 0.22$ ,  $m_t^{\text{phys}} = 174 \text{ GeV}$  and  $Y_t/Y_f = 0.98$  as input values, we obtain  $m_d(1 \text{ GeV}) = 7.7 \text{ MeV}$ ,  $m_s(1 \text{ GeV}) = 193 \text{ MeV}$ ,  $m_b(m_b) = 4.26 \text{ GeV}$ ,  $|V_{cb}| = 0.050$ ,  $|V_{ub}|/|V_{cb}| = 0.059$ ,  $J = 2.96 \times 10^{-5}$ . The value of  $|V_{cb}|$  corresponding to  $\alpha_s(M_Z) = 0.125$ ,  $m_c(m_c) = 1.22 \text{ GeV}$ ,  $m_t^{\text{phys}} = 190 \text{ GeV}$  is  $|V_{cb}| = 0.045$ . In the case of three Yukawa unification,  $h_t = h_b = h_\tau$ ,  $|V_{cb}|$  is unrenormalized and results in a larger value, which is therefore disfavored. Note that all the predictions of the GJ ansatz are presently in good agreement with experiments, especially for the case of small  $\tan\beta$ .

We now turn to the derivation of Eq. (1), which is our main result. Notice that the up and the down quark mass matrices in Eq. (1) have a very asymmetric structure. We will show how to generate such an up-down asymmetry within  $SO(10)$ . First we observe that all elements of the fermion mass matrices must arise from effective scalar operators that transform as **10**, **120** or  $\overline{\mathbf{126}}$  of  $SO(10)$ , each of which can couple to the fermion bilinears. The **10** and  $\overline{\mathbf{126}}$  operators result in symmetric mass matrices. In order to obtain an up-down asymmetry, the effective **10** or  $\overline{\mathbf{126}}$  operator that generates the (23) element of  $M_u$  must develop an electroweak VEV only along the up (and not along the down) direction. Similarly, the  $\overline{\mathbf{126}}$  operator that induces the (22) element of  $M_d$  and  $M_l$  must develop an electroweak VEV only along the down direction. Now, if the successful prediction of  $\sin^2\theta_W$  of SUSY GUTs is to be preserved, only one pair of Higgs(ino) doublets can remain

light ( $H_u$  and  $H_d$  of MSSM). These are the only doublets that acquire electroweak VEVs. There could be several doublet Higgs(ino)s at the GUT scale which mix with one another, but in such a way that only one pair of them remains light [15]. The texture of the mass matrices in Eq. (1) then requires that the Higgs doublet coupling to the (23) entry in  $M_u$  should have an admixture of  $H_u$ , but not of  $H_d$  and similarly, the Higgs doublet generating (22) entry of  $M_d$  and  $M_l$  should have an admixture of  $H_d$ , but not  $H_u$ . This leads to the important observation that the texture zeros of the fermion mass matrices can be ensured only if the Higgs(ino) mass matrix has its own texture zeros. Furthermore, it requires that the Higgs(ino) mass matrix be asymmetric, otherwise an up–down asymmetry cannot be induced.

Which linear combination of up–type and down–type Higgs(ino) doublets remains light is of course related to the question of doublet–triplet splitting of SUSY GUTs. The simplest way to achieve a natural doublet–triplet splitting in  $SO(10)$  is via the Dimopoulos–Wilczek mechanism[16]. This is achieved by the coupling  $H_1 A H_2$  where  $H_1$  and  $H_2$  belong to the **10** of  $SO(10)$ , while  $A$  is the adjoint **45**. This term gives masses to the color triplets in **10**’s but not to the  $SU(2)$  doublets if the VEV of the adjoint  $A$  is chosen to be along the  $(B - L)$  direction:  $\langle A \rangle = \text{diag.}(a, a, a, 0, 0) \times i\tau_2$ . In realistic  $SO(10)$  models, this pattern of vev’s is not stable and new techniques are needed to make this method useful [17,18]. Furthermore, it leads to up–down symmetric mass matrix textures for the fermions, which are not useful for our purpose.

In order to generate an asymmetry in the Higgs(ino) mass matrix we propose to mix the **10**-plets ( $H_1$  and  $H_2$ ) with the spinorial **16** +  $\overline{\mathbf{16}}$  ( $\psi_H + \overline{\psi}_H$ ), which are needed anyway for symmetry breaking. Each **10** contains in it one up–type and one down–type Higgs doublet,  $\psi_H$  contains a down–type doublet, while  $\overline{\psi}_H$  contains an up–type doublet. We have therefore a total of three up–type and three down–type Higgs doublets. The **10**’s also contain a color triplet–antitriplet pair,  $\psi_H$  has a color antitriplet while  $\overline{\psi}_H$  has the color triplet. The superpotential involving the  $SU(2)$  doublets and the color triplets is assumed to be

$$W_{DT} = \lambda_1 \psi_H \psi_H H_1 + \lambda_2 \overline{\psi}_H \overline{\psi}_H H_2 + H_1 A H_2 . \quad (6)$$

This is the most general superpotential relevant for the doublet–triplet splitting compatible with the set of discrete symmetries given in Table. 1. Eq. (6) leads to the following mass matrices for the Higgs(ino) doublets and color triplets (written in the basis where the rows correspond to  $(\psi_{H_d}, H_{2d}, H_{1d})$  and the columns stand for  $(\overline{\psi}_{H_u}, H_{1u}, H_{2u})$  for the doublets and similarly for the color triplets)

$$M_D = \begin{pmatrix} 0 & \lambda_1 v_R & 0 \\ \lambda_2 v_R & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad M_T = \begin{pmatrix} 0 & \lambda_1 v_R & 0 \\ \lambda_2 v_R & \lambda_3 a & 0 \\ 0 & 0 & -\lambda_3 a \end{pmatrix} . \quad (7)$$

Here  $\langle\psi_H\rangle = \langle\bar{\psi}_H\rangle = v_R$  and  $\langle A\rangle = a$ . This gives GUT scale Dirac masses to all three triplet–antitriplet pairs. Two pairs of Higgs(ino) doublets become superheavy, leaving one pair of doublets light. The  $H_u$  and  $H_d$  fields of MSSM are easily identified as  $H_u = H_{2u}$  and  $H_d = H_{1d}$ . Note that the desired asymmetry has been achieved,  $H_1$  will acquire a VEV only along the down direction ( $\langle H_{1d}\rangle \neq 0, \langle H_{1u}\rangle = 0$ ), while  $H_2$  will acquire a VEV only along the up direction ( $\langle H_{2d}\rangle = 0, \langle H_{2u}\rangle \neq 0$ ). This serves as the first step in our derivation of the GJ ansatz. In this scheme,  $\tan\beta \neq m_t/m_b$  owing to the **(10-16)** mixing.

Let us now turn to the symmetry breaking sector. Nonrenormalizable operators suppressed by appropriate powers of inverse Planck mass, which are expected to arise in superstring theories, will play a crucial role in our analysis. In addition, we shall make use of a few gauge singlet superfields to generate the GUT scale starting from the Planck scale and the SUSY breaking scale  $m_0 \sim 1 \text{ TeV}$ . To see how this works, consider a nonrenormalizable superpotential involving a singlet superfield

$$W = \lambda \frac{\phi^n}{M_{Pl}^{n-3}} \quad (8)$$

where  $n$  is an integer. The corresponding scalar potential, including soft supersymmetry breaking terms is given by

$$V = m_0^2 |\phi|^2 + \left| \frac{n\lambda\phi^{n-1}}{M_{Pl}^{n-3}} \right|^2 + m_0 A_n \left[ \frac{\lambda\phi^n}{M_{Pl}^{n-3}} + H.C. \right] \quad (9)$$

In the supersymmetric limit, the potential has a unique minimum given by  $\langle\phi\rangle = 0$ . But including SUSY breaking, the potential develops two other minima given by

$$\langle|\phi|\rangle = M_{Pl} \left[ \frac{1}{2} \left| \frac{A_n m_0}{\lambda M_{Pl} n(n-1)} \right| \left( 1 \pm \sqrt{1 - \frac{4(n-1)}{A_n^2}} \right) \right]^{\frac{1}{n-2}}. \quad (10)$$

For  $n = 8, 9, 10$ , this leads to  $\langle|\phi|\rangle \sim 10^{16} \text{ GeV}$  which is near the GUT scale. The existence of these symmetry breaking minima requires that  $A_n^2 \geq 4(n-1)$  (see Eq. (10)). We have investigated if this constraint is satisfied automatically in  $N = 1$  supergravity models with hidden sector SUSY breaking *a la* Polony. We found  $|A_n| = |n - \sqrt{3}|$  in this scheme, which satisfies the constraint for  $n \geq 7$ .

Let us now display the part of the superpotential that breaks the gauge symmetry down to the standard model. A minimum of one adjoint **45** and a **16+16** pair

Higgs superfields are required for this [17,18]. In order to generate the GJ texture, we shall also need a second adjoint  $A'$ .  $A$ , which has a VEV along the  $(B-L)$  direction,  $\langle A \rangle = \text{diag.}(a, a, a, 0, 0) \times i\tau_2$ , is responsible for the doublet-triplet splitting.  $A'$  develops a VEV along the  $I_{3R}$  direction,  $\langle A' \rangle = \text{diag.}(0, 0, 0, a', a') \times i\tau_2$ . In addition, we shall need a few singlets, in order to generate the GUT scale as discussed above, as well as for the fermion mass matrix texture. These singlets are denoted by  $X, Y, Z, \bar{Z}$  and  $S_{1,2,3}$ , all of which develop GUT scale VEVs except for the field  $X$  which has zero VEV. The superpotential which achieves the symmetry breaking and the desired VEV structure is given by

$$W_{sym} = X(\bar{\psi}_H \psi_H - Y^2) + A^2 Z^2 + A^2 \bar{Z}^2 + A^4 + A^2 A'^2 + A'^2 Z^2 + A'^2 \bar{Z}^2 + A'^4 + (Z\bar{Z})^2 + Z^4 + \bar{Z}^4 + M\bar{Z}Z + S_1^8 + S_2^8 + S_1^2 S_2^4 S_3^4 + X^8 A^2 + \dots \quad (11)$$

This is the most general superpotential up to relevant orders consistent with the discrete symmetries given in Table 1. We should mention here that we have not tried to be economical with the choice of the discrete symmetry. Rather, we have identified the maximal symmetry of the desired superpotential in Table 1. The full symmetry of Table 1 may not be essential to achieve GJ texture. It is understood that in Eq. (11), each term is suppressed by the dimensionally appropriate powers of  $M_{Pl}$ . Terms such as  $A^4, A^2 A'^2$  in Eq. (11) stand for more than one possible contraction of indices. If the mass parameter  $M$  is chosen to be  $M_{GUT}^2/M_{Pl}$ ,  $\langle A \rangle, \langle A' \rangle \sim M_{GUT}$  will be generated. The superpotential gives rise to the VEVs of  $A$  and  $A'$  in the desired  $(B-L)$  and  $I_{3R}$  directions respectively. Except for the question of linking the  $(A, A')$  sector to the  $(\psi_H, \bar{\psi}_H)$  sector (this issue will be addressed shortly), which is necessary to avoid pseudogoldstone bosons, the symmetry breaking down to the standard model has now been achieved. As for the corrections to the doublet Higgs(ino) mass matrix, the leading order terms are  $H_1 H_2 Y^4/M_{Pl}^3$  and  $H_1 H_2 (\psi_H \bar{\psi}_H)^2/M_{Pl}^3$ . These will induce an effective  $\mu$  parameter for  $H_u$  and  $H_d$  of MSSM. Demanding that the  $\mu$  parameter is  $\lesssim 1 \text{ TeV}$ , we find that  $\langle \psi_H \rangle, \langle Y \rangle \sim 10^{15} \text{ GeV}$ , which is satisfactorily close to the GUT scale. Terms such as  $\psi_H \psi_H H_2$  that can also contribute to the  $\mu$  parameter do not arise until very high order.

We now proceed to the derivation of the GJ texture. The relevant superpotential involving the matter fields consistent with the symmetries of Table 1 is

$$W_{Yuk} = M^{-1} (h_{33} \psi_3 \psi_3 H_1 S_1 + h'_{33} \psi_3 \psi_3 H_2 S_2 + h_{23} \psi_2 \psi_3 H_2 S_3) + M^{-2} h_{22} \psi_2 \psi_2 H_1 A A' + M^{-3} (h_{12} \psi_1 \psi_2 H_1 S_2^3 + h'_{12} \psi_1 \psi_2 H_2 S_1^3) . \quad (12)$$

Here  $M$  is not necessarily the Planck mass, Eq. (12) could arise by integrating out vector-like fermions such as  $\bar{\mathbf{16}} + \mathbf{16}$  which have an intermediate mass between the GUT and the Planck scale. All parameters in Eq. (12) can be made real by field



redefinitions, but the VEVs of  $A, A'$  are complex in general, so one unremovable phase will reappear after symmetry breaking as in Eq. (1).

The  $h_{22}$  term in Eq. (13) has quite interesting properties, which needs a bit of explanation. In general, the  $H_1 AA'$  can give rise to effective **10** as well as  $\overline{\mathbf{126}}$  operators. The GJ texture requires that only the  $\overline{\mathbf{126}}$  contributes. The interesting feature of this term with  $\langle A \rangle$  along  $(B-L)$  direction and  $\langle A' \rangle$  along  $I_{3R}$  direction is that only the  $\overline{\mathbf{126}}$  operator contributes to the mass matrices. Effective **10** can arise in three different ways, all of which vanish at the minimum, due to the orthogonality of  $\langle A \rangle$  and  $\langle A' \rangle$ . To see this, we first observe that  $\text{Tr}(AA') = 0$ , which sets one such contribution to zero. Other terms of the type  $A_{ab}A'_{bc}H_{1c}$  and  $A'_{ab}A_{bc}H_{1c}$  also vanish at the minimum. One is left with the term  $A_{ab}A'_{cd}H_e$ , which is precisely the  $\overline{\mathbf{126}}$  contribution. This does not vanish and as a result, we are able to reproduce the relative factor of  $-3$  in the (22) entry of  $M_d$  and  $M_l$ .

If all the Yukawa coupling parameters in Eq. (12) are chosen to be of order one, the hierarchy in the masses can be explained purely by the ratio  $\langle S_i \rangle / M \equiv \epsilon$ . Except for  $S_2$ , which generates the top quark mass, we will choose these ratios to be in the range  $1/10$  to  $1/30$ . As noted already, the doublet-triplet splitting mechanism implies that  $H_{1d} = H_d$  and  $H_{2u} = H_u$ . Making this identification, we see that below the GUT scale, Eq. (12) results in the following effective Yukawa superpotential (with redefined  $h_{ij}$ ):

$$\begin{aligned} W_{Yuk}^{eff} = & h'_{33}Q_3H_uu_3^c + \epsilon h_{33}(Q_3H_d d_3^c + L_3H_d e_3^c) + h_{23}\epsilon(Q_2H_uu_3^c + Q_3H_uu_2^c) + \\ & h_{22}\epsilon^2(Q_2H_d d_2^c - 3L_2H_d e_2^c) + h'_{12}\epsilon^3(Q_1H_uu_2^c + Q_2H_uu_1^c) + \\ & h_{12}\epsilon^3(Q_1H_d d_2^c + Q_2H_d d_1^c + L_1H_d e_2^c + L_2H_d e_1^c) . \end{aligned} \quad (13)$$

The desired GJ texture follows from this after electro-weak symmetry breaking. It is easy to see that the texture zeros of the mass matrices are protected to very high order by the discrete symmetry.

Let us now turn briefly to the neutrino sector. The Dirac neutrino matrix is identical to the up quark matrix in the model. As for the right-handed neutrino Majorana mass matrix, we have no more freedom to choose its form since its discrete symmetry assignment has already been fixed in the process of deriving the GJ ansatz. The allowed right-handed neutrino mass terms are

$$W_{\nu_R} = \left( M^{-3}\psi_3\psi_3S_1A + M^{-2}\psi_2\psi_2A' + M^{-10}\psi_1\psi_1A'^3S_2^6 \right) \overline{\psi}_H\overline{\psi}_H . \quad (14)$$

The resulting  $\nu_R$  mass matrix is nonsingular, and thus generates small neutrino masses for  $\nu_e, \nu_\mu$  and  $\nu_\tau$ . Note that although the last term in Eq. (14) has higher inverse powers of  $M$ , it is compensated largely by the VEV of  $S_2$ , which cannot be too much below  $M$  as it generates the top quark mass. Thus the neutrino spectrum is

similar to what is expected in non-SUSY  $SO(10)$  models with an intermediate scale. Such a spectrum is known to be capable of resolving the solar neutrino puzzle via the MSW mechanism while leaving the  $\nu_\tau$  mass in the cosmologically significant multi-eV range. It is interesting that the superstring constraint on the Higgs spectrum is what is responsible for the weakening of the see-saw suppressions somewhat.

Let us turn now to the question of linking the  $(A, A')$  and the  $(\bar{\psi}_H, \psi_H)$  sectors. If the two sectors were not linked, there will be pseudogoldstones belonging to a  $\mathbf{10} + \overline{\mathbf{10}}$  of  $SU(5)$ . Although the successful prediction of  $\sin^2\theta_W$  will be unaffected, since they form complete multiplets of  $SU(5)$ , they do upset the prediction for  $m_b$ . In order to make them superheavy, one cannot link the two sectors directly, as that would upset the VEV of  $A$ . The simplest way to achieve this is to assume a term  $\text{Tr}(AA'A'')$  along with  $A''\bar{\psi}_H\psi_H$  in the superpotential. Here  $A''$  is another adjoint. The  $\text{Tr}(AA'A'')$  term, due to its complete anti-symmetry, does not upset the VEV structure of  $A, A'$ , yet it makes all the pseudogoldstones superheavy. The introduction of a third adjoint would appear to be in conflict with the superstring constraints, however,  $A''$  need not survive below the Planck scale, in which case there is no contradiction. To be specific, let us add the following terms to the superpotential:

$$W' = \bar{\psi}_H\psi_H A'' P + AA'A''Q + (A''Q)^2 + Q\bar{Q} + (Q\bar{Q})^2 + (P\bar{Q})^8 \quad (15)$$

where  $P, Q, \bar{Q}$  are gauge singlets. These terms are clearly consistent with the discrete symmetries of Table 1.  $W'$  also has a  $Z_n$  symmetry under which  $(A'', \bar{Q})$  and  $(P, Q)$  have opposite charges. This superpotential admits  $\langle Q \rangle \sim M_{Pl}$ , so that  $A''$  has a mass of order  $M_{Pl}$ .  $P$  gets a VEV of order  $M_{GUT}$  from the last term in Eq. (15). The coupling of  $A''$  to the other fields make all pseudogoldstones heavy, of order  $M_{GUT}^2/M_{Pl}$ .  $A''$  will receive an induced VEV, but it is of order  $M_{GUT}^3/M_{Pl}^2$ . The only effect of  $A''$  on the fermion mass matrices is to give a tiny correction to the (22) element of  $M_{d,l}$ , via the term  $\psi_2\psi_2 H_1 A'' Q$ , which is about  $10^{-3}$  times smaller than the leading term.

Let us finally show how the method developed here facilitates the derivation of other symmetric mass matrix textures. Take for example, Model (4) of Ref. (11), which is obtained by adding a (22) entry in  $M_u$  of Eq. (1). Such a texture follows readily in our scheme by a new Yukawa term  $\psi_2\psi_2 H_2 S$  with  $S$  a gauge singlet carrying discrete charge of  $(4, 4, 0, 2)$ . All the five symmetric texture models of Ref. (11) can be derived in an analogous fashion. Our method can be applied to derive asymmetric texture models as well in the context of  $SO(10)$  [19].

Let us conclude by observing some interesting variations of the doublet-triplet splitting scheme (Eqs. (6)-(7)). One could add to Eq. (6) direct mass terms  $(M_\psi\psi_H\bar{\psi}_H + M_1 H_1^2 + M_2 H_2^2)$  with  $M_\psi M_1 M_2 = 0$  to ensure a light doublet. If  $M_1 = 0$ ,

the MSSM fields  $H_u$  and  $H_d$  are  $H_u \propto (M_\psi M_2 H_{1u} + \lambda_1 \lambda_2 v_R^2 H_{2u} - \lambda_1 v_R M_2 \bar{\psi}_{H_u})$  and  $H_d = H_{1d}$ . Thus  $\langle H_{1u} \rangle, \langle H_{1d} \rangle, \langle H_{2u} \rangle, \langle \bar{\psi}_{H_u} \rangle \neq 0$ , while  $\langle H_{2d} \rangle = \langle \bar{\psi}_{H_d} \rangle = 0$ . Such a spectrum enables one to use Yukawa couplings to the  $H_1$  field to induce some common elements in  $M_{u,d,l}$  while generating an up-down asymmetry via the couplings of  $H_2$  and  $\bar{\psi}_{H_u}$ . If  $M_\psi = 0, M_1 M_2 \neq 0$ , then  $H_d = \cos\theta H_{1d} + \sin\theta \psi_{H_d}$  and  $H_u = \cos\theta' H_{2u} + \sin\theta' \bar{\psi}_{H_u}$ , with  $\theta, \theta'$  Higgs(ino) mixing angles. The up-down asymmetry is similar to Eq. (7), but now there is the freedom of involving the  $\psi_{H_d}, \bar{\psi}_{H_u}$  fields in the fermion mass generation. Details of these and application to other textures will be the subject of a forthcoming publication.

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**Table. 1.** The matter superfields along with their discrete charge assignments. The subscript in  $Z$  and  $\bar{Z}$  stand for a  $Z_2$  symmetry under which all other fields are even.

Superfield	$Z_8 \times Z_{16} \times Z_{16} \times Z_4$
$\psi_H(\mathbf{16})$	(1,0,0,0)
$\bar{\psi}_H(\overline{\mathbf{16}})$	(7,2,0,0)
$H_1(\mathbf{10})$	(6,0,0,0)
$H_2(\mathbf{10})$	(2,12,0,0)
$A(\mathbf{45})$	(0,4,0,0)
$A'(\mathbf{45})$	(0,12,0,2)
$Y(\mathbf{1})$	(0,1,0,0)
$X(\mathbf{1})$	(0,14,0,0)
$Z(\mathbf{1})$	(0,4,0,0) <sub>-</sub>
$\bar{Z}(\mathbf{1})$	(0,12,0,0) <sub>-</sub>
$\psi_1(\mathbf{16})$	(7,4,6,1)
$\psi_2(\mathbf{16})$	(1,0,0,1)
$\psi_3(\mathbf{16})$	(0,0,1,1)
$S_1(\mathbf{1})$	(2,0,14,2)
$S_2(\mathbf{1})$	(14,4,14,2)
$S_3(\mathbf{1})$	(13,4,15,2)